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POLYNOMIALS YIELDING QUADRUPLES WITH PROPERTY D(k)

A.M.S. Ramasamy

Department of Mathematics,
Vel Tech Rangarajan Dr. Sagunthala R&D
Institute of Science and Technology, Avadi, Chennai - 600062, INDIA

E-mail: ramasamyams@veltech.edu.in

Dedicated to Prof. A.K. Agarwal on his 70^{th} ***Birth Anniversary***

Abstract: Let k be a natural number. Two integers α and β are said to have the property D(k) (resp. D(-k)) if $\alpha\beta$+k (resp. $\alpha\beta-$k) is a perfect square. The purpose of this paper is identification of polynomials producing quadruples with property D(k) for certain values of k. Incidentally the paper brings out an attribute of Ramanujan number 1729 in contributing two quadruples of polynomials with property D(k).

Keyword and Phrases: Property p_k, extendable set, $P_{r,k}$ sequence, Pell's equation, quadruple with Diophantine property, Ramanujan number.

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1. Introduction

The Greek mathematician Diophantus raised the question as to four numbers such that the product of any two increased by a given number shall be a square. M.Gardner [11] asked for a fifth number that can be added to the set $\{1, 3, 8, 120\}$ without destroying the property that the product of any two integers is one less than a perfect square. For historical details of the problem, one may refer to J.Roberts [24] and the author [19].

It is seen that the polynomials x, x+2, 4x+4 have the property that the product of any two of them increased by 1 is a square. A fourth polynomial that works with these three is $16x^3 + 48x^2 + 44x + 12$. B.W.Jones [12, 13] considered polynomials for this problem. He found all polynomials that work with x and x+2. He defined